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LETTER TO THE EDITOR

Path integral for new coherent states of Lie superalgebra *osp* (1/2, *R*)

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Abstract. A path integral formulation in the representation of new coherent states for the Lie superalgebra $osp(1/2, R)$ is introduced. By the use of the completeness relation of the new coherent states, a path integral expression for the transition amplitude between two $osp(1/2, R)$ new coherent states for a Hamiltonian which is linear in the generators of the superalgebra are obtained. In the classical limit the equations of motion for the system are derived.

It is well known that both path integral [1] and coherent states (CSs) [2] have played major roles in the study of quantum mechanical systems, particularly for establishing the correspondence between classical and quantum physics. Radcliffe [3] first proposed CS for the $SU(2)$ group, then Perelomov [4] and Gilmore [5] generalized the construction of CSs to arbitrary Lie groups. In the past few years, much attention has been paid to the study of the generalized CSs for Lie superalgebras [6–11].

The use of CSs to provide an alternative method of obtaining the phase-space path integral, and hence the Hamilton equation of motion, was first proposed by Klauder [12]. This path integral technique has been extended to include a formulation in terms of generalized CSs for many other Lie (super)algebras [8, 13, 14]. The CS path integral formalism has also found application in the study of Berry's phase [15] and the Jaynes–Cummings model in quantum optics [16, 17]. In the past decade, there have been hints of physically realized supersymmetry in quantum optics [18], nuclear [19] and many-body physics [20]. Therefore it is a worthwhile effort to study the path integral for Lie superalgebras.

As is well known, the Lie superalgebra $osp(1/2, R)$ is one of the most important and the simplest Lie superalgebras. The authors of [6, 7] introduced $osp(1/2, R)$ CSs by using Perelomov's definition of CSs for arbitrary groups [4]. In [21], starting with the boson realization of the $osp(1/2, R)$ superalgebra, we presented a new kind of CSs for the $osp(1/2, R)$, and studied the D-algebra differential realization [22] of the $osp(1/2, R)$ in the new CS representation. One of the interesting features of the $osp(1/2, R)$ new CSs is that they can encompass the Glauber CSs, $su(1, 1)$ CSs and squeezed states in quantum optics within a common formalism. In this letter, we intend to study the path integral formalism of the $osp(1/2, R)$ new CSs. In section 2, we present a brief summary of the results for the $osp(1/2, R)$ new CSs. In section 3 we study the path integral formulation of the transition amplitude between two $osp(1/2, R)$ new CSs. Section 4 is devoted to deriving the classical equations of motion for the system.

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We now consider new coherent states for $osp(1/2, R)$ superalgebra. The Lie superalgebra $osp(1/2, R)$ has five generators which satisfy the following commutation and anticommutation relations:

$$[K_0, K_\pm] = \pm K_\pm \qquad [K_+, K_-] = -2K_0 \tag{1}$$

$$[K_0, F_\pm] = \pm\tfrac{1}{2}F_\pm \qquad [K_\pm, F_\pm] = 0 \qquad [K_\pm, F_\mp] = \mp F_\pm \tag{2}$$

$$\{F_\pm, F_\pm\} = K_\pm \qquad \{F_+, F_-\} = K_0 \tag{3}$$

which contains a subalgebra $su(1, 1)$ spanned by K_+, K_- and K_0 as its even part.

It is easy to check that these $osp(1/2, R)$ generators admit the following boson realization:

$$K_+ = \tfrac{1}{2}a^{\dagger 2} \qquad K_- = \tfrac{1}{2}a^2 \qquad K_0 = \tfrac{1}{2}(a^\dagger a + \tfrac{1}{2}) \tag{4}$$

$$F_+ = \tfrac{1}{2}a^\dagger \qquad F_- = \tfrac{1}{2}a \tag{5}$$

where $[a, a^\dagger] = 1$, and a and $a^\dagger$ are the annihilation and creation operators of a boson, respectively.

Making use of (4) and (5), one can obtain the number representation of the superalgebra:

$$K_+|n\rangle = \tfrac{1}{2}\sqrt{(n+1)(n+2)}|n+2\rangle \qquad K_-|n\rangle = \tfrac{1}{2}\sqrt{(n(n-1)}|n-2\rangle \tag{6}$$

$$K_0|n\rangle = \tfrac{1}{2}(n+\tfrac{1}{2})|n\rangle \qquad F_+|n\rangle = \tfrac{1}{2}\sqrt{n+1}|n+1\rangle \qquad F_-|n\rangle = \tfrac{1}{2}\sqrt{n}|n-1\rangle. \tag{7}$$

A new CS of the $osp(1/2, R)$ is defined as

$$|\alpha\beta\rangle = S(\beta)D(\alpha)|0\rangle \tag{8}$$

where α and β are two arbitrary complex numbers, and the operators $S(\beta)$ and $D(\alpha)$ are defined by

$$S(\beta) = \exp(\beta K_+ - \beta^* K_-) \qquad D(\alpha) = \exp(\alpha F_+ - \alpha^* F_-) \tag{9}$$

which correspond to even and odd parts of the superalgebra, respectively.

$S(\beta)$ and $D(\alpha)$ satisfy the following commutation relation:

$$D(\alpha)S(\beta) = S(\beta)D(\alpha\cosh r + \alpha^* \mathrm{e}^{\mathrm{i}\theta}\sinh r) \qquad \beta = r\mathrm{e}^{\mathrm{i}\theta}. \tag{10}$$

From (4) to (8), one can arrive at the number-state representation of the new CSs,

$$|\alpha\beta\rangle = \sum_{n=0}^{\infty}(n!\cosh r)^{-1/2}(\tfrac{1}{2}\mathrm{e}^{\mathrm{i}\theta}\tanh r)^{n/2}\exp[-\tfrac{1}{8}(|\alpha|^2 - \alpha^2\mathrm{e}^{\mathrm{i}\theta}\tanh r)] \times H_n\left[\frac{\alpha}{2}(\mathrm{e}^{\mathrm{i}\theta}\sinh r)^{-1/2}\right]|n\rangle \tag{11}$$

where $H_n(x)$ is the Hermite polynomial.

These CSs are normalized, i.e., $\langle\alpha\beta \mid \alpha\beta\rangle = 1$, they have the following orthogonality relation:

$$\langle\alpha'\beta' \mid \alpha\beta\rangle = A^{1/2}(\beta,\beta')\exp\left\{-\tfrac{1}{8}(|\alpha|^2+|\alpha'|^2)+\tfrac{1}{2}A(\beta,\beta')[B(\beta,\beta')\alpha^2 + 2\alpha\alpha'^* + B^*(\beta,\beta')\alpha^{*2}]\right\} \tag{12}$$

with

$$A(\beta,\beta') = (\cosh r\cosh r' - \mathrm{e}^{\mathrm{i}(\theta-\theta')}\sinh r\sinh r')^{-1} \tag{13}$$

$$B(\beta,\beta') = \mathrm{e}^{-\mathrm{i}\theta}\sinh r\cosh r' - \mathrm{e}^{-\mathrm{i}\theta'}\cosh r\sinh r'. \tag{14}$$

These CSs form an overcomplete Hilbert space, it can be proved that they have the following completeness relation,

$$\int \mathrm{d}^2\alpha\,\mathrm{d}^2\beta\,\sigma(\alpha,\beta)|\alpha\beta\rangle\langle\alpha\beta| = \sum_{n=0}^{\infty}|n\rangle\langle n| = 1 \tag{15}$$

where the weight function $\sigma(\alpha,\beta)$ is given by

$$\sigma(\alpha,\beta) = \frac{1}{\pi}\delta(\operatorname{Re}\beta)\delta(\operatorname{Im}\beta). \tag{16}$$

Let us consider a Hamiltonian H acting in a Hilbert space. We shall assume that the Hamiltonian can be expanded as a finite polynomial of the generators of the $osp(1/2, R)$ superalgebra. The time evolution of the quantum mechanical system is determined by the evolution operator:

$$U(t',t) = \boldsymbol{T}\exp\left\{\int_t^{t'}\left[-\frac{\mathrm{i}}{\hbar}H(\tau)\,\mathrm{d}\tau\right]\right\} \tag{17}$$

where $\boldsymbol{T}$ is the Dyson time-ordering operator.

Following the standard approach, one can factorize the evolution operator as follows:

$$\begin{aligned} U(t',t) &= \lim_{N\to,\epsilon\to 0}\exp\left[-\frac{\mathrm{i}}{\hbar}H(\tau_N)\epsilon\right]\cdots\exp\left[-\frac{\mathrm{i}}{\hbar}H(t_j)\epsilon\right]\cdots\exp[-\mathrm{i}\hbar H(t)\epsilon] \\ &= \lim_{N\to,\epsilon\to 0}\prod_{j=1}^{N}U_\epsilon(\tau_j) \end{aligned} \tag{18}$$

where

$$U_\epsilon(\tau_j) = \exp\left[-\frac{\mathrm{i}}{\hbar}\epsilon H(\tau_j)\right] \qquad \epsilon = \frac{t'-t}{N}. \tag{19}$$

The transition amplitude (or propagator) from the coherent state $|\alpha\beta\rangle$ at time t to the coherent state $|\alpha'\beta'\rangle$ at time t' is given by

$$K(\alpha',\beta',t',\alpha,\beta,t) = \langle\alpha',\beta'|U(t',t)|\alpha\beta\rangle \tag{20}$$

with the following defining properties,

$$\Psi(\alpha_{t'},\beta_{t'}) = \int \mathrm{d}\alpha\,\mathrm{d}\beta\,\sigma(\alpha,\beta)K(\alpha'\beta',t';\alpha,\beta,t)\Psi(\alpha_t,\beta_t). \tag{21}$$

Making use of the resolution of unity of the new CSs N times, substituting (18) into (20) and inserting the completeness relation (15) into each time interval, we can rewrite the transition amplitude as follows:

$$\begin{aligned}K(\alpha',\beta',t',\alpha,\beta,t) &= \lim_{N\to\infty,\epsilon\to 0}\int\cdots\prod_{j=1}^{N}\mathrm{d}^2\alpha_j\,\mathrm{d}^2\beta_j\,\sigma(\alpha_j,\beta_j)\\ &\quad\times\prod_{j=1}^{N+1}\langle\alpha_j\beta_j|\left[1-\frac{\mathrm{i}}{\hbar}\epsilon H(t_j)\right]|\alpha_{j-1}\beta_{j-1}\rangle\\ &= \lim_{N\to\infty,\epsilon\to 0}\int\cdots\int\prod_{j=1}^{N}\mathrm{d}^2\alpha_j\,\mathrm{d}^2\beta_j\,\sigma(\alpha_j,\beta_j)\\ &\quad\times\exp\left\{\sum_{j=1}^{N+1}\left[\ln\langle\alpha_j\beta_j\mid\alpha_{j-1}\beta_{j-1}\rangle-\frac{\mathrm{i}}{\hbar}\epsilon\mathcal{H}(\alpha_j,\beta_j;\alpha_{j-1},\beta_{j-1})\right]\right\}\end{aligned} \tag{22}$$

where we have used the symbol,

$$\mathcal{H}(\alpha_j,\beta_j;\alpha_{j-1},\beta_{j-1}) \equiv \frac{\langle\alpha_j,\beta_j|H|\alpha_{j-1}\beta_{j-1}\rangle}{\langle\alpha_j,\beta_j\mid\alpha_{j-1},\beta_{j-1}\rangle}. \tag{23}$$

Through a tedious but straightforward evaluation, the first term in the exponent on the right-hand side of equation (22) can be expressed as

$$\begin{aligned}\ln\langle\alpha_j,\beta_j\mid\alpha_{j-1}\beta_j-1\rangle &= \tfrac{1}{2}(\alpha_j\Delta\alpha_j^*-\alpha_j^*\Delta\alpha_j-\mathrm{i}\sinh^2 r_j\Delta\theta_j)\\ &\quad-\tfrac{1}{2}(2\mathrm{e}^{\mathrm{i}\theta}\Delta r_j+\mathrm{i}\sinh r_j\cosh r_j\mathrm{e}^{\mathrm{i}\theta_j}\Delta\theta_j)\alpha_j^{*2}\\ &\quad+\tfrac{1}{2}(2\Delta r_j\mathrm{e}^{-\mathrm{i}\theta_j}-\mathrm{i}\sinh r_j\cosh r_j\mathrm{e}^{-\mathrm{i}\theta_j}\Delta\theta_j)\alpha_j^2\end{aligned} \tag{24}$$

where $\Delta\alpha_j\equiv\alpha_j-\alpha_{j-1}$, $\Delta\alpha_j^*\equiv\alpha_j^*-\alpha_{j-1}^*$ and $\Delta\theta_j\equiv\theta_j-\theta_{j-1}$, and we have taken $\beta_j=r_j\mathrm{e}^{\mathrm{i}\theta_j}$.

Substituting (24) into (22), we can obtain the final expression for the transition amplitude which can be written as the formal functional integral:

$$K(\alpha',\beta',t';\alpha,\beta,t) = \int \mathrm{D}^2\alpha\mathrm{D}^2\beta\exp\left(\frac{\mathrm{i}}{\hbar}S\right) \tag{25}$$

with the following action

$$S = \int_t^{t'}\mathcal{L}(\alpha(\tau),\dot\alpha(\tau),\alpha^*(\tau),\dot\alpha^*(\tau),r(\tau),\dot r(\tau),\theta(\tau),\dot\theta(\tau))\,\mathrm{d}\tau \tag{26}$$

where the Lagrangian is defined by

$$\mathcal{L} = \frac{\mathrm{i}\hbar}{2}(\dot\alpha\alpha^*-\alpha\dot\alpha^*)-\frac{\hbar}{2}\sinh^2 r\dot\theta+\hbar(\alpha^{*2}\mathrm{e}^{\mathrm{i}\theta}-\alpha^2\mathrm{e}^{-\mathrm{i}\theta})(\mathrm{i}\dot r-\tfrac{1}{2}\sinh r\cosh r\dot\theta)-\mathcal{H}(\alpha,\beta). \tag{27}$$

Next we investigate the classical dynamics of the system governed by the Hamiltonian H. The classical limit is now understood as the case in which $\hbar$ is extremely small compared with the action S in equation (26), the main contribution to the transition amplitude comes from the path which makes the action stationary with fixed endpoint $\alpha = \alpha(t)$, $\alpha' = \alpha'(t')$, $r = r(t)$, $r' = r(t')$, $\theta = \theta(t)$ and $\theta' = \theta(t')$:

$$\begin{aligned} 0 = \delta S = \int_t^{t'} \Bigg\{ & \left[\frac{\partial \mathcal{L}}{\partial \alpha} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}}\right)\right]\delta\alpha + \left[\frac{\partial \mathcal{L}}{\partial \alpha^*} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}^*}\right)\right]\delta\alpha^* \\ & + \left[\frac{\partial \mathcal{L}}{\partial r} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right)\right]\delta r + \left[\frac{\partial \mathcal{L}}{\partial \theta} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right)\right]\delta\theta \Bigg\}\, \mathrm{d}\tau. \end{aligned} \tag{28}$$

As the variations $\delta\alpha$, $\delta\alpha^*$, δr and $\delta\theta$ are independent and arbitrary, we obtain the Euler–Lagrange equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}}\right) - \frac{\partial \mathcal{L}}{\partial \alpha} = 0 \tag{29}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\alpha}^*}\right) - \frac{\partial \mathcal{L}}{\partial \alpha^*} = 0 \tag{30}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) - \frac{\partial \mathcal{L}}{\partial r} = 0 \tag{31}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0. \tag{32}$$

Using the expression (27), we can rewrite the above equations of motion as:

$$\dot{\alpha} = -\alpha^*(\dot{r} - \mathrm{i}\dot{\theta}\sinh r\cosh r)\mathrm{e}^{-\mathrm{i}\theta} - \frac{\mathrm{i}}{\hbar}\left(\frac{\partial \mathcal{H}}{\partial \alpha^*}\right) \tag{33}$$

$$\dot{\alpha}^* = -\alpha(\dot{r} + \mathrm{i}\dot{\theta}\sinh r\cosh r)\mathrm{e}^{\mathrm{i}\theta} + \frac{\mathrm{i}}{\hbar}\left(\frac{\partial \mathcal{H}}{\partial \alpha}\right) \tag{34}$$

$$\dot{r} = \frac{2\,\mathrm{Im}(\alpha\dot{\alpha}\mathrm{e}^{\mathrm{i}\theta}) + (\mathrm{i}\dot{\theta} + \dot{\theta}^2\sinh r\cosh r)\,\mathrm{Im}(\alpha^2\mathrm{e}^{-\mathrm{i}\theta}) - (1/\hbar)(\partial\mathcal{H}/\partial\theta)}{\mathrm{i}\,\mathrm{Im}(\alpha^2\mathrm{e}^{-\mathrm{i}\theta}) - \sinh r\cosh r} \tag{35}$$

$$\dot{\theta} = \frac{2\mathrm{i}\,\mathrm{Im}(\alpha\dot{\alpha}\mathrm{e}^{-\mathrm{i}\theta}) - (1/\hbar)(\partial\mathcal{H}/\partial r)}{\sinh r\cosh r - \alpha^2\mathrm{e}^{\mathrm{i}\theta}}. \tag{36}$$

In particular, when $r = 0$ and $\theta = 0$, the above equations reduce to

$$\dot{\alpha} = -\frac{\mathrm{i}}{\hbar}\frac{\partial \mathcal{H}}{\partial \alpha^*} \tag{37}$$

$$\dot{\alpha}^* = \frac{\mathrm{i}}{\hbar}\frac{\partial \mathcal{H}}{\partial \alpha} \tag{38}$$

which are just the classical dynamical equations of the system for Glauber coherent states [16].

When one takes $\alpha = 0$, equations (35) and (36) become

$$\dot{\theta} = -\frac{1}{\hbar \sinh r \cosh r} \frac{\partial \mathcal{H}}{\partial r} \tag{39}$$

$$\dot{r} = \frac{1}{\hbar \sinh r \cosh r} \frac{\partial \mathcal{H}}{\partial \theta} \tag{40}$$

which are the classical dynamical equations of the system for the $su(1,1)$ coherent states from the boson realization (4).

We now consider the classical dynamical equations of motion (33) to (36) for the most general Hamiltonian that belongs to the $osp(1/2, R)$ superalgebra with time-dependent coefficients,

$$H(t) = A(t)K_0 + f_1(t)K_+ + f_1^*(t)K_- + f_2(t)F_+ + f_2^*(t)F_-. \tag{41}$$

This Hamiltonian is useful in quantum physics. In fact, by using the boson realization of the $osp(1/2, R)$ generators, one can express the above Hamiltonian as

$$H(t) = \tfrac{1}{2}A(t)a^\dagger a + \tfrac{1}{2}f_1(t)a^{\dagger 2} + \tfrac{1}{2}f_1^*(t)a^2 + \tfrac{1}{2}f_2 a^\dagger + \tfrac{1}{2}f_2^* a + \tfrac{1}{4}A(t) \tag{42}$$

which is the most general coherence-preserving Hamiltonian in quantum optics.

For the Hamiltonian (41), we have

$$\begin{aligned} \mathcal{H}(\alpha, \beta) = {} & \tfrac{1}{2}A(t)[\sinh^2 r + \alpha\alpha^* \cosh 2r + 2\,\mathrm{Re}(\alpha^{*2}\mathrm{e}^{\mathrm{i}\theta}) \sinh r \cosh r + \tfrac{1}{2}] \\ & + \mathrm{Re}\{f_1(t)[\alpha^2 \mathrm{e}^{-\mathrm{i}2\theta} \sinh^2 r + \alpha^{*2} \cosh^2 r + (1 + 2\alpha\alpha^*) \sinh r \cosh r \mathrm{e}^{-\mathrm{i}\theta}]\} \\ & + \mathrm{Re}[f_2(t)(\alpha^* \cosh r + \alpha \mathrm{e}^{-\mathrm{i}\theta} \sinh r)]. \end{aligned} \tag{43}$$

In the derivation of the above equation, we have used the following mean values:

$$\begin{aligned} \langle \alpha\beta | K_+ | \alpha\beta \rangle &= \langle \alpha\beta | K_- | \alpha\beta \rangle^* \\ &= \tfrac{1}{2}[\alpha^{*2} \cosh^2 2r + \alpha^2 \mathrm{e}^{-\mathrm{i}2\theta} \sinh^2 2r + (1 + 2\alpha\alpha^*)\mathrm{e}^{-\mathrm{i}\theta} \sinh 2r \cosh 2r] \end{aligned} \tag{44}$$

$$\begin{aligned} \langle \alpha\beta | K_0 | \alpha\beta \rangle = {} & \tfrac{1}{4} + \tfrac{1}{2}[(1 + \alpha\alpha^*) \sinh^2 2r + \alpha\alpha^* \cosh^2 2r \\ & + 2\,\mathrm{Re}(\alpha^2 \mathrm{e}^{-\mathrm{i}\theta}) \sinh 2r \cosh 2r] \end{aligned} \tag{45}$$

$$\langle \alpha\beta | F_+ | \alpha\beta \rangle = \langle \alpha\beta | F_- | \alpha\beta \rangle^* = \tfrac{1}{2}(\alpha^* \cosh 2r + \alpha \mathrm{e}^{-\mathrm{i}\theta} \sinh 2r). \tag{46}$$

It follows from equation (43) that

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \alpha} = {} & \tfrac{1}{2}A(\alpha^* \cosh 2r + 2\alpha \mathrm{e}^{-\mathrm{i}\theta} \sinh r \cosh r) + 2\alpha^* \,\mathrm{Re}(f_1 \mathrm{e}^{-\mathrm{i}\theta}) \sinh r \cosh r + \alpha f_1 \mathrm{e}^{-2\mathrm{i}\theta} \sinh^2 r \\ & + \alpha f_1^* \cosh^2 r + \tfrac{1}{2} f_2 \mathrm{e}^{-\mathrm{i}\theta} \sinh r + \tfrac{1}{2} f_2^* \cosh r \end{aligned} \tag{47}$$

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \alpha^*} = {} & \tfrac{1}{2}A(\alpha \cosh 2r + 2\alpha^* \mathrm{e}^{\mathrm{i}\theta} \sinh r \cosh r) + 2\alpha \,\mathrm{Re}(f_1 \mathrm{e}^{-\mathrm{i}\theta}) \sinh r \cosh r + \alpha^* f_1^* \mathrm{e}^{2\mathrm{i}\theta} \sinh^2 r \\ & + \alpha^* f_1 \cosh^2 r + \tfrac{1}{2} f_2^* \mathrm{e}^{\mathrm{i}\theta} \sinh r + \tfrac{1}{2} f_2 \cosh r \end{aligned} \tag{48}$$

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial r} = {} & A[\sinh r \cosh r + \mathrm{Re}(\alpha^{*2} \mathrm{e}^{\mathrm{i}\theta}) \cosh 2r] + \mathrm{Re}[2(\alpha^{*2} + \alpha^2 \mathrm{e}^{-\mathrm{i}2\theta}) f_1 \sinh r \cosh r \\ & + f_1(1 + 2\alpha\alpha^*)\mathrm{e}^{-\mathrm{i}\theta} \cosh 2r + \alpha f_2 \mathrm{e}^{-\mathrm{i}\theta} \cosh r + \alpha^* f_2 \sinh r] \end{aligned} \tag{49}$$

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial \theta} = {} & A\,\mathrm{Im}(\alpha^2 \mathrm{e}^{-\mathrm{i}\theta}) \sinh r \cosh r + \mathrm{Im}[\alpha f_2 \mathrm{e}^{-\mathrm{i}\theta} \sinh r + 2\alpha^2 f_1 \mathrm{e}^{-\mathrm{i}2\theta} \sinh^2 r \\ & + (1 + 2\alpha\alpha^*) f_1 \mathrm{e}^{-\mathrm{i}\theta} \sinh r \cosh r]. \end{aligned} \tag{50}$$

Substituting (47)–(50) into (33)–(36), one can obtain the classical equations of motion of the system governed by the Hamiltonian (41). Obviously, these classical equations of motion are complicated nonlinear differential equations.

In conclusion we have introduced a path integral formalism in the representation of new CSs for the Lie superalgebra and derived the classical equations of motion for the system in which time evolution was driven by a coherence-preserving Hamiltonian. It has been shown that these classical equations of motion are highly nonlinear, and their forms follow directly from the fact that the CSs are constructed from a Lie superalgebra. The classical equations of motion for systems associated with the Glauber CSs and $su(1,1)$ CSs from boson realization can be obtained as special cases of the $osp(1/2, R)$ formulation. These show that it is possible to apply the results obtained in this letter to some problems in quantum mechanics and quantum optics.

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